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RESEARCH
REPORT
N° 8949
August 2016
Project-Team Socrate

ISSN 0249-6399   ISRN INRIA/RR--8949--FR+ENG

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Research Report n° 8949 — August 2016 — 20 pages

Abstract: In this report, the fundamental limits of *decentralized* information transmission in the K-user Gaussian multiple access channel (G-MAC), with $K \geqslant 2$, are fully characterized. Two scenarios are considered. First, a game in which only the transmitters are players is studied. In this game, the transmitters autonomously and independently tune their own transmit configurations seeking to maximize their own information transmission rates, $R_1, R_2, \ldots, R_K$, respectively. On the other hand, the receiver adopts a fixed receive configuration that is known a priori to the transmitters. The main result consists of the full characterization of the set of rate tuples $(R_1, R_2, \ldots, R_K)$ that are achievable and stable in the G-MAC when stability is considered in the sense of the η-Nash equilibrium (NE), with $\eta \geqslant 0$ arbitrarily small. Second, a sequential game in which the two categories of players (the transmitters and the receiver) play in a given order is presented. For this sequential game, the main result consists of the full characterization of the set of rate tuples $(R_1, R_2, \ldots, R_K)$ that are stable in the sense of an η-sequential equilibrium, with $\eta \geqslant 0$.

Key-words: Stability, capacity, Gaussian, multiple-access channel, Nash equilibrium, sequential equilibrium

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Transmission décentralisée et simultanée d'information et d'énergie dans les canaux à accès multiple

Résumé : Dans le présent-rapport, les limites fondamentales de la transmission décentralisée d'information dans le canal Gaussien à accès multiple (G-MAC) à K utilisateurs où $K \geqslant 2$ sont déterminées. Deux scénarios sont considérés. Premièrement, un jeu où seuls les transmetteurs jouent est étudié. Dans ce jeu, les transmetteurs règlent leurs configurations d'émission d'une manière autonome et indépendante dans le but de maximiser leurs débits individuels de transmission d'information $R_1, \ldots, R_K$, respectivement. En contrepartie, le récepteur adopte une configuration de réception qui est fixe et connue au préalable de tous les transmetteurs. Le résultat principal est la caractérisation de l'ensemble des débits $(R_1, R_2, \ldots, R_K)$ atteignables et stables dans le G-MAC quand la stabilité est considérée au sens du η-équilibre de Nash, pour un $\eta \geqslant 0$ arbitrairement petit. Deuxièmement, un jeu séquentiel est présenté dans lequel deux catégories de joueurs jouent dans un ordre donné. Pour ce jeu séquentiel, le résultat principal est la caractérisation de l'ensemble des débits $(R_1, R_2, \ldots, R_K)$ qui sont stables au sens d'un η-équilibre séquentiel, pour un $\eta \geqslant 0$ arbitrairement petit.

Mots-clés : Stabilité, région de capacité, canal Gaussien à accès multiple, équilibre de Nash, équilibre séquentiel.

Contents

1 Problem Formulation

This section describes first the channel model, the K-user (centralized) Gaussian Multiple Access Channel (G-MAC) and then provides an game-theoretic formulation of the decentralized information transmission in this channel.

1.1 K-User Centralized Gaussian Multiple Access Channel

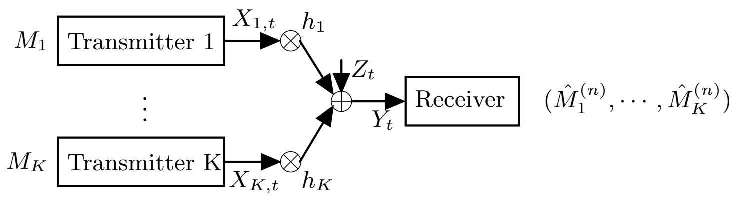

Figure 1: K-user memoryless Gaussian MAC.

Consider the K-user memoryless G-MAC with $K \geqslant 2$ users as shown in Fig. 1. Let $n \in \mathbb{N}$ be the blocklength. At each time $t \in \{1, 2, \ldots, n\}$ and for any $i \in \{1, 2, \ldots, K\}$, let $X_{i,t}$ denote the real input symbol sent by transmitter i. The receiver observes the real channel output

$$Y_t = \sum_{i=1}^{K} h_i X_{i,t} + Z_t, \tag{1}$$

where h_i, for all $i \in \{1, 2, \ldots, K\}$, is a constant nonnegative real channel coefficient. The noise terms Z_t are independent and identically distributed realizations of a zero-mean unit-variance real Gaussian random variable.

Let R_i denote the information transmission rate at transmitter i, for all $i \in \{1, 2, \ldots, K\}$. The goal of the communication is to convey the message index M_i, uniformly distributed over the set $\mathcal{M}_i \triangleq \{1, 2, \ldots, \lfloor 2^{nR_i} \rfloor\}$, from transmitter i, with $i \in \{1, 2, \ldots, K\}$ to the common receiver. The message indices $(M_1, M_2, \ldots, M_K)$ are independent of each other and of the noise terms $Z_1, Z_2, \ldots, Z_n$.

At each time t, the t-th symbol of transmitter i, for all $i \in \{1, 2, \ldots, K\}$, depends solely on its message index M_i, i.e.,

$$X_{i,t} = f_{i,t}^{(n)}(M_i), \quad t \in \{1, 2, \ldots, n\}, \tag{2}$$

for some encoding functions $f_{i,t}^{(n)} \colon \mathcal{M}_i \to \mathbb{R}$.

The receiver produces an estimate $(\hat{M}_1^{(n)}, \hat{M}_2^{(n)}, \ldots, \hat{M}_K^{(n)}) = \Phi^{(n)}(Y_1, Y_2, \ldots, Y_n)$ of the message-tuple $(M_1, M_2, \ldots, M_K)$ via a decoding function $\Phi^{(n)} \colon \mathbb{R}^n \to \mathcal{M}_1 \times \mathcal{M}_2 \times \cdots \times \mathcal{M}_K$, and the average probability of error is given by

$$P_{\text{error}}^{(n)}(R_1, R_2, \ldots, R_K) \triangleq \Pr\left\{(\hat{M}_1^{(n)}, \hat{M}_2^{(n)}, \ldots, \hat{M}_K^{(n)}) \neq (M_1, M_2, \ldots, M_K)\right\}. \tag{3}$$

The symbols $X_{i,1}, X_{i,2}, \ldots, X_{i,n}$ satisfy an expected average *input power constraint*

$$\frac{1}{n} \sum_{t=1}^{n} \mathrm{E}\left[X_{i,t}^2\right] \leqslant P_{i,\max}, \quad i \in \{1, 2, \ldots, K\}, \tag{4}$$

where the expectation is over the message indices and where $P_{i,\max}$ denotes the maximum average power of transmitter i in energy units per channel use.

This channel is fully described by the signal to noise ratios (SNRs): SNR_i, with $i \in \{1, 2, \ldots, K\}$, which are defined as follows:

$$\mathrm{SNR}_i \triangleq |h_i|^2 P_{i,\max}. \tag{5}$$

1.2 Achievable Rates and Capacity Region

The K-tuple $(R_1, R_2, \ldots, R_K) \in \mathbb{R}_+^K$ is said to be *achievable* if there exists a sequence of encoding and decoding functions $\left\{\{f_{1,t}^{(n)}\}_{t=1}^n, \{f_{2,t}^{(n)}\}_{t=1}^n, \ldots, \{f_{K,t}^{(n)}\}_{t=1}^n, \Phi^{(n)}\right\}_{n=1}^{\infty}$ such that the average error probability tends to zero as the blocklength n tends to infinity. That is,

$$\limsup_{n\to\infty} \; P_{\mathrm{error}}^{(n)}(R_1, R_2, \ldots, R_K){=}0. \tag{6}$$

The closure of the union of all achievable rate tuples is called the *capacity region* and is denoted by $\mathcal{C}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$. From [1, 2], it follows that

$$\mathcal{C}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) = \\ \left\{(R_1, R_2, \ldots, R_K) \in \mathbb{R}_+^K : \sum_{j\in\mathcal{U}} R_j \leqslant \frac{1}{2}\log_2\left(1 + \sum_{j\in\mathcal{U}} \mathrm{SNR}_j\right), \forall \mathcal{U} \subseteq \{1, 2, \ldots, K\}\right\}. \tag{7}$$

Note that $\mathcal{C}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ is a K-dimension polyhedron with $K!$ corner points. Each corner point corresponds to a decoding order among the users.

1.3 K-User Decentralized Gaussian Multiple Access Channel

In a decentralized K-user G-MAC, the aim of transmitter i, for all $i \in \{1, 2, \ldots, K\}$, is to autonomously choose its transmit configuration s_i in order to maximize its information rate R_i. The transmit configuration s_i can be described in terms of the information rates R_i, the block-length n, the channel input alphabet $\mathcal{X}_i$, the encoding functions $\{f_{1,t}^{(n)}\}_{t=1}^n, \{f_{2,t}^{(n)}\}_{t=1}^n, \ldots, \{f_{K,t}^{(n)}\}_{t=1}^n$, etc. The receiver autonomously chooses a receive configuration s_0 in view of maximizing the sum-rate. Let $\mathcal{P}_K$ denote the set of all permutations (all possible decoding orders) over the set $\{1, 2, \ldots, K\}$. For any $\pi \in \mathcal{P}_K$, the considered decoding order $\pi(1), \pi(2), \ldots, \pi(K)$ is such that user $\pi(1)$ is decoded first, user $\pi(2)$ is decoded second, etc. The receive configuration can be described in terms of the decoding function $\Phi^{(n)}$, which in this report is restricted to single-user decoding (SUD), successive interference cancelation (SIC(π)) with a given order $\pi \in \mathcal{P}_K$, or any time-sharing (TS) combination of the previous schemes. However, the choice of the transmit configuration of each transmitter depends on the choice of the other transmitters as well as the decoding scheme at the receiver. The input signal of one transmitter is interference to the others. Thus, the rate achieved by transmitter i depends on all transmit configurations $s_1, s_2, \ldots, s_K$ as well as the configuration of the receiver s_0.

The utility function of transmitter i, for all $i \in \{1, 2, \ldots, K\}$, is $u_i : \mathcal{A}_0 \times \mathcal{A}_1 \times \cdots \times \mathcal{A}_K \to \mathbb{R}_+$ and it is defined as its own rate,

$$u_i(s_0, s_1, \ldots, s_K) = \begin{cases} R_i(s_0, s_1, \ldots, s_K), & \text{if} \quad P_{\mathrm{error}}^{(n)}(R_1, R_2, \ldots, R_K) < \epsilon \\ 0, & \quad\quad \text{otherwise}, \end{cases} \tag{8}$$

where $\epsilon > 0$ is an arbitrarily small number and $R_i(s_0, s_1, \ldots, s_K)$ denotes a transmission rate achievable with the configurations $(s_0, s_1, \ldots, s_K)$. Often, the information rate $R_i(s_0, s_1, \ldots, s_K)$

is written as R_i for simplicity. However, every nonnegative achievable information rate is associated with a particular transmit-receive configuration $(s_0, s_1, \ldots, s_K)$ that achieves it. It is worth noting that there might exist several transmit-receive configurations that achieve the same tuple $(R_1, R_2, \ldots, R_K)$ and distinction between the different transmit-receive configurations is made only when needed.

The utility function of the receiver is $u_0 : \mathcal{A}_0 \times \mathcal{A}_1 \times \cdots \times \mathcal{A}_K \to \mathbb{R}_+$ and it is defined as the sum-rate,

$$u_0(s_0, s_1, \ldots, s_K) = \begin{cases} \sum_{i=1}^{K} R_i(s_0, s_1, \ldots, s_K), & \text{if} \quad P_{\text{error}}^{(n)}(R_1, R_2, \ldots, R_K) < \epsilon \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

In the absence of a central controller which dictates the transmit and receive configurations to the various network components, only stable rate tuples are possible operating points of the network. Within this context, stability is considered in the sense that none of the network components is able to increase its utility by unilaterally changing its own transmit/receive configuration. From this perspective, in the capacity region $\mathcal{C}(\text{SNR}_1, \text{SNR}_2, \ldots, \text{SNR}_K)$, any rate tuple $(R_1, R_2, \ldots, R_K)$ for which

$$R_i < \frac{1}{2}\log_2\left(1 + \frac{\text{SNR}_i}{1 + \sum_{j=1;j\neq i}^{K} \text{SNR}_j}\right), \tag{10}$$

at least for one $i \in \{1, 2, \ldots, K\}$ is not stable. This is true when the receiver is constrained to choose among the decoding strategies mentioned above (SUD, SIC, or TS) because the considered transmitter can always increase its information rate and achieve

$$R_i = \frac{1}{2}\log_2\left(1 + \frac{\text{SNR}_i}{1 + \sum_{j=1;j\neq i}^{K} \text{SNR}_j}\right) - \delta, \tag{11}$$

with $\delta > 0$ arbitrarily small.

The remaining achievable rate tuples $(R_1, R_2, \ldots, R_K) \in \mathcal{C}(\text{SNR}_1, \text{SNR}_2, \ldots, \text{SNR}_K)$ which satisfy

$$R_i \geqslant \frac{1}{2}\log_2\left(1 + \frac{\text{SNR}_i}{1 + \sum_{j=1;j\neq i}^{K} \text{SNR}_j}\right), \quad \forall i \in \{1, 2, \ldots, K\}, \tag{12}$$

can be stable or not, depending on the actions of the receiver.

In the following, two games are considered. First, a game in which only the transmitters are players is studied in Sec. 2. For this game, the set of stable rate tuples is fully characterized when stability is considered in the sense of η-Nash equilibrium [3], with $\eta \geqslant 0$ arbitrarily small. Second, a sequential game in which the two categories of players (the transmitters and the receiver) play in a given order. For this sequential game, the set of stable rate tuples in the sense of the η-sequential equilibrium, with $\eta \geqslant 0$ arbitrarily small, is derived in Sec. 3.

2 Game I: Only the Transmitters are Players

Under the assumption that the receiver adopts a fixed receive configuration $\tilde{s}_0$ that is known a priori to all terminals, the competitive interaction of the K transmitters in the decentralized G-MAC can be modeled by the following game in normal form:

$$\mathcal{G}_1 = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{u_k\}_{k\in\mathcal{K}}\right). \tag{13}$$

The set $\mathcal{K} = \{1, 2, \ldots, K\}$ is the set of players, i.e., the transmitters. For all $i \in \mathcal{K}$, the set $\mathcal{A}_i$ is the set of actions of player i. An action $s_i \in \mathcal{A}_i$ of player i is basically its transmit configuration as described above. The utility function of transmitter i, for all $i \in \{1, 2, \ldots, K\}$, is u_i defined in (8). Note that since the receiver is not a player, its action $\tilde{s}_0$ is kept fixed, but it remains being an argument of the utility function.

A formal definition of an η-NE is provided below.

Definition 1 (η-NE [3])**.** *In the game $\mathcal{G}_1$, under the fixed receive configuration $\tilde{s}_0$, an action profile $(\tilde{s}_0, s_1^*, \ldots, s_K^*)$ is an η-NE if for all $i \in \mathcal{K}$ and for all $s_i \in \mathcal{A}_i$, it holds that*

$$u_i(\tilde{s}_0, s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_K^*) \leqslant u_i(\tilde{s}_0, s_1^*, \ldots, s_{i-1}^*, s_i^*, s_{i+1}^*, \ldots, s_K^*) + \eta. \qquad (14)$$

Under the fixed receive configuration $\tilde{s}_0$, from Def. 1, it becomes clear that if $(\tilde{s}_0, s_1^*, \ldots, s_K^*)$ is an η-NE, then none of the transmitters can increase its own rate by more than η bits per channel use by unilaterally changing its own transmit configuration while keeping the average error probability arbitrarily close to zero. Thus, at a given η-NE, every transmitter achieves a utility that is η-close to its maximum achievable rate given the transmit configuration of the other transmitters. Note that if $\eta = 0$, then the definition of NE is obtained [4].

Remark 1. *Note that the definition of the utilities in (8) and (9) is parametrized by the choice of the error probability threshold ϵ. Within this context, considering NE instead of η-NE with an arbitrary slack $\eta \geqslant 0$ would require the difficult task of deriving a coding scheme that achieves the optimal rate with exactly ϵ error probability. The slack $\eta \geqslant 0$, which can be made arbitrarily small, allows to remove this difficulty [5] and [6]. Note also that there is a slight abuse of notation in the equalities defining the utilities. Here it is assumed that the blocklength is chosen to be sufficiently high to neglect the asymptotically small slack due to the fixed blocklength.*

The following investigates the rate region that can be achieved at an η-NE. This set of rate tuples is known as the η-NE region.

Definition 2 (η-NE Region)**.** *Let $\eta \geqslant 0$ be arbitrarily small. An achievable rate tuple $(R_1, R_2, \ldots, R_K) \in \mathcal{C}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ is said to be in the η-NE region of the game $\mathcal{G}_1 = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{u_k\}_{k\in\mathcal{K}}\right)$ under the fixed receive configuration $\tilde{s}_0$, if there exists an action profile $(\tilde{s}_0, s_1^*, \ldots, s_K^*) \in \mathcal{A}_0 \times \mathcal{A}_1 \times \cdots \times \mathcal{A}_K$ that is an η-NE and the following holds:*

$$u_i(\tilde{s}_0, s_1^*, \ldots, s_K^*) = R_i, \quad \forall i \in \{1, 2, \ldots, K\}. \qquad (15)$$

The following section studies the η-NE region of the game $\mathcal{G}_1$, with $\eta \geqslant 0$ arbitrarily small, for several decoding strategies adopted by the receiver.

2.1 η-NE Region With Single User Decoding (SUD)

The η-NE region of the game $\mathcal{G}_1$ when the receiver uses SUD, denoted by $\mathcal{N}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$, is described by the following theorem.

Theorem 1 (η-NE Region With SUD)**.** *Let $\eta \geqslant 0$ be arbitrarily small. Then, the set $\mathcal{N}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ of η-NEs of the game $\mathcal{G}_1$ contains only the nonnegative rate tuple $(R_1, R_2, \ldots, R_K)$ that satisfies*

$$R_i = \frac{1}{2}\log_2\left(1 + \frac{\mathrm{SNR}_i}{1 + \sum_{j=1; j\neq i}^{K} \mathrm{SNR}_j}\right), \forall i \in \{1, 2, \ldots, K\}. \qquad (16)$$

Proof: The proof of Theorem 1 is provided in Appendix A. ■

2.2 η-NE Region With Successive Interference Cancelation (SIC)

The η-NE region of the game $\mathcal{G}_1$ when the receiver uses SIC(π) with a fixed decoding order $\pi \in \mathcal{P}_K$, denoted by $\mathcal{N}_{\mathrm{SIC}(\pi)}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$, is described by the following theorem.

Theorem 2 (η-NE Region of the Game $\mathcal{G}_1$ With SIC(π))**.** *Let $\eta \geqslant 0$ be arbitrarily small and let $\pi \in \mathcal{P}_K$ be a fixed decoding order. Then, the set $\mathcal{N}_{\mathrm{SIC}(\pi)}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ contains only the nonnegative rate tuple $(R_1, R_2, \ldots, R_K)$ satisfying:*

$$R_{\pi(i)} = \frac{1}{2} \log_2 \left(1 + \frac{\mathrm{SNR}_{\pi(i)}}{1 + \sum_{j=i+1}^{K} \mathrm{SNR}_{\pi(j)}} \right), \forall i \in \{1, 2, \ldots, K\}. \tag{17}$$

Proof: The proof of Theorem 2 is provided in Appendix B. ■

Remark 2. *Note that for every decoding order $\pi \in \mathcal{P}_K$, the region contains a unique rate tuple. When considering SIC at the receiver under any decoding order, the η-NE region $\mathcal{N}_{\mathrm{SIC}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ is given by*

$$\mathcal{N}_{\mathrm{SIC}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) = \bigcup_{\pi \in \mathcal{P}_K} \mathcal{N}_{\mathrm{SIC}(\pi)}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K), \tag{18}$$

and it contains $K!$ rate tuples.

2.3 η-NE Region With Time-Sharing (TS)

Let $\mathcal{N}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ denote the η-NE region of the game $\mathcal{G}_1$ when the receiver might use any time-sharing between the previous decoding techniques. This region is described by the following theorem.

Theorem 3 (η-NE Region of the Game $\mathcal{G}_1$)**.** *Let $\eta \geqslant 0$ be arbitrarily small. Then, the set $\mathcal{N}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ is*

$$\begin{aligned}&\mathcal{N}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) = \\ &\text{Conv. hull} \left(\mathcal{N}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \cup \left(\bigcup_{\pi \in \mathcal{P}_K} \mathcal{N}_{\mathrm{SIC}(\pi)}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \right) \right).\end{aligned} \tag{19}$$

Proof: The proof is based on Theorem 1, Theorem 2, and a time-sharing argument. The details are omitted. ■

If the receiver performs any time-sharing combination between any of the considered decoding strategies, then the transmitters can use the same time-sharing combination between their corresponding η-NE strategies to achieve any point inside $\mathcal{N}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$. Note that every time-sharing strategy of the receiver induces a unique rate tuple inside $\mathcal{N}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$. However, several time-sharing schemes might achieve the same rate tuple.

3 Game II: A Sequential Game

In this section, the decentralized information transmission in the K-user G-MAC is modeled as a sequential game in which there are two groups of players: one group, the leaders, in which all players play simultaneously before the players of the other group, the followers. The followers, simultaneously play after the leaders under the assumption that the actions of the leaders are perfectly known by all the followers. Let $\{\mathcal{K}_{21}, \mathcal{K}_{22}\}$ be a partition of $\mathcal{K} \cup \{0\}$, such that $\mathcal{K}_{21}$ is the set of leaders and $\mathcal{K}_{22}$ is the set of followers.

The competition between the different users (the transmitters and the receiver) in the G-MAC can be modeled as follows:

$$\mathcal{G}_2 = \left(\mathcal{K} \cup \{0\}, \{\mathcal{K}_{21}, \mathcal{K}_{22}\}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{u_k\}_{k\in\mathcal{K}}\right). \tag{20}$$

Backward induction is used in order to characterize a *sequential equilibrium* of this game, . First, the leaders simultaneously play knowing that the followers will simultaneously play their *best responses.*

In the game $\mathcal{G}_2$, instead of seeking an exactly optimal solution, each player allows a *tolerance* $\eta \geqslant 0$ and seeks a strategy that is η-close to the optimal reward. The set of these η-close optimal strategies of player k is given by its best η-response set defined as follows:

Definition 3 (Set of Best η-Response of Player k)**.** *For a given player $k \in \{0, 1, \dots, K\}$, the set of best η-responses is*

$$\mathrm{BR}_k^{(\eta)}(\mathbf{s}_{-k}) = \left\{ s_k \in \mathcal{A}_k : u_k(s_k, \mathbf{s}_{-k}) \geqslant \max_{\tilde{s}_k \in \mathcal{A}_k} u_k(\tilde{s}_k, \mathbf{s}_{-k}) - \eta \right\}. \tag{21}$$

Note that even when $\eta = 0$, the best η-response set does not necessarily reduce to a unitary set.

Definition 4 (η-Sequential Equilibrium (η-SE))**.** *Let $\eta \geqslant 0$ be arbitrarily small. In the game $\mathcal{G}_2$, an action profile $(s_0^\dagger, s_1^\dagger, , \dots, s_K^\dagger)$ is an η-SE if it satisfies the following two conditions:*

1. $\forall i \in \mathcal{K}_{21}$, $s_i^\dagger \in \mathrm{BR}_i^{(\eta)}\left(\mathbf{s}^\dagger_{\mathcal{K}_{21}\setminus\{i\}}\right)$ *with*

$$\begin{aligned}\mathrm{BR}_i^{(\eta)}\left(\mathbf{s}^\dagger_{\mathcal{K}_{21}\setminus\{i\}}\right) \triangleq \Bigg\{ & s_i \in \mathcal{A}_i : u_i(s_i, \mathbf{s}^\dagger_{\mathcal{K}_{21}\setminus\{i\}}, \mathbf{s}_{\mathcal{K}_{22}}) \geqslant \max_{\tilde{s}_i \in \mathcal{A}_i} u_i(\tilde{s}_i, \mathbf{s}^\dagger_{\mathcal{K}_{21}\setminus\{i\}}, \tilde{\mathbf{s}}_{\mathcal{K}_{22}}) - \eta \\ & \text{subject to} \quad \mathbf{s}_{\mathcal{K}_{22}} \in \mathrm{BR}^{(\eta)}_{\mathcal{K}_{22}}\left(s_i, \mathbf{s}^\dagger_{\mathcal{K}_{21}\setminus\{i\}}\right) \text{ and } \tilde{\mathbf{s}}_{\mathcal{K}_{22}} \in \mathrm{BR}^{(\eta)}_{\mathcal{K}_{22}}\left(\tilde{s}_i, \mathbf{s}^\dagger_{\mathcal{K}_{21}\setminus\{i\}}\right) \Bigg\},\end{aligned}$$

with

$$\mathrm{BR}^{(\eta)}_{\mathcal{K}_{22}}\left(s_i, \mathbf{s}^\dagger_{\mathcal{K}_{21}\setminus\{i\}}\right) \triangleq \prod_{j\in\mathcal{K}_{22}} \mathrm{BR}_j^{(\eta)}\left(\mathbf{s}_{\mathcal{K}_{22}\setminus\{j\}}, s_i, \mathbf{s}^\dagger_{\mathcal{K}_{21}\setminus\{i\}}\right).$$

2. $\forall j \in \mathcal{K}_{22}, \quad s_j^\dagger \in \mathrm{BR}_j^{(\eta)}\left(\mathbf{s}^\dagger_{\mathcal{K}_{22}\setminus\{j\}}, \mathbf{s}^\dagger_{\mathcal{K}_{21}}\right).$

Note that when $\eta = 0$ and when for all the action profile $\mathbf{s}_{\mathcal{K}_{21}} \in \mathcal{A}_{\mathcal{K}_{21}}$ of the leaders, the set $\mathrm{BR}^{(0)}_{\mathcal{K}_{22}}(\mathbf{s}_{\mathcal{K}_{21}})$ is unitary, the definition of a Stackelberg equilibrium [7] is obtained. Note also that the η-sequential equilibrium in Def. 4 can be seen as a generalization of the sequential Stackelberg equilibrium concept presented in [8] for two-person games and it results in a two-stage η-NE. That is, a first η-NE is established among the leaders under the assumption that the followers are playing their η-best responses. A second η-NE is observed among the followers under the assumption that the actions played by the leaders are perfectly known by the followers.

Definition 5 (η-Sequential Equilibrium Region)**.** *An achievable rate tuple $(R_1, R_2, \ldots, R_K)$ is said to be in the η-SE region of the game $\mathcal{G}_2$, if there exists an action profile $(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger) \in \mathcal{A}_0 \times \mathcal{A}_1 \times \cdots \times \mathcal{A}_K$ that is an η-SE and such that*

$$u_i(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger) = R_i, \quad \forall i \in \{1, 2, \ldots, K\}, \tag{22}$$

$$u_0(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger) = \sum_{i=1}^{K} R_i. \tag{23}$$

3.1 η-Sequential Equilibrium Region With the Receiver as a Leader

Consider the game in which the receiver chooses first a receive configuration (is the leader) and the transmitters adapt their transmit configurations to the choice of the decoding rule in order to maximize their utilities (are the followers), i.e., $\mathcal{K}_{21} = \{0\}$ and $\mathcal{K}_{22} = \{1, 2, \ldots, K\}$.

Let $\mathcal{S}_R(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ denote the η-SE region of the game $\mathcal{G}_2$ when the receiver is the leader. This region is described by the following theorem.

Theorem 4 (η-SE Region of the Game $\mathcal{G}_2$ With the Receiver as a Leader)**.** *The set $\mathcal{S}_R(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ contains all nonnegative rate tuples $(R_1, R_2, \ldots, R_K)$ satisfying*

$$\sum_{i=1}^{K} R_i = \frac{1}{2}\log_2\left(1 + \sum_{i=1}^{K} \mathrm{SNR}_i\right). \tag{24}$$

Proof: The proof of Theorem 4 is provided in Appendix C. ■

3.2 η-Sequential Equilibrium Region With Transmitter i as a Leader

Consider the game in which transmitter i, for a given $i \in \{1, 2, \ldots, K\}$, chooses first its transmit configuration and the receiver and the remaining transmitters follow, i.e., $\mathcal{K}_{21} = \{i\}$ and $\mathcal{K}_{22} = \{0, 1, \ldots, K\} \setminus \{i\}$. Let $\eta \geqslant 0$ be arbitrarily small and let $\mathcal{S}_i(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ denote the η-SE region of the game $\mathcal{G}_2$ when the transmitter i is the leader. This region is described by the following theorem.

Theorem 5 (η-SE Region of the Game $\mathcal{G}_2$ With Transmitter i as a Leader)**.** *The set $\mathcal{S}_i(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ contains all nonnegative rate tuples $(R_1, R_2, \ldots, R_K)$ satisfying*

$$R_i = \frac{1}{2}\log_2\left(1 + \mathrm{SNR}_i\right), \tag{25}$$

$$\sum_{j=1; j\neq i}^{K} R_j = \frac{1}{2}\log_2\left(1 + \sum_{j=1}^{K} \mathrm{SNR}_j\right) - \frac{1}{2}\log_2\left(1 + \mathrm{SNR}_i\right). \tag{26}$$

Proof: The proof of Theorem 5 is provided in Appendix D. ■

4 Example and Observations

In the two-user G-MAC, the regions described in Theorems 1-5 are illustrated in Fig. 2, with the capacity region plotted as a reference. Namely, the η-Nash equilibrium regions $\mathcal{N}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2)$, $\mathcal{N}_{\mathrm{SIC}(\pi_1)}(\mathrm{SNR}_1, \mathrm{SNR}_2)$, $\mathcal{N}_{\mathrm{SIC}(\pi_2)}(\mathrm{SNR}_1, \mathrm{SNR}_2)$, and $\mathcal{N}(\mathrm{SNR}_1, \mathrm{SNR}_2)$ in Theorems 1-3 are plotted in red, with π_i the decoding order in which transmitter i is decoded first, for all $i \in \{1, 2\}$.

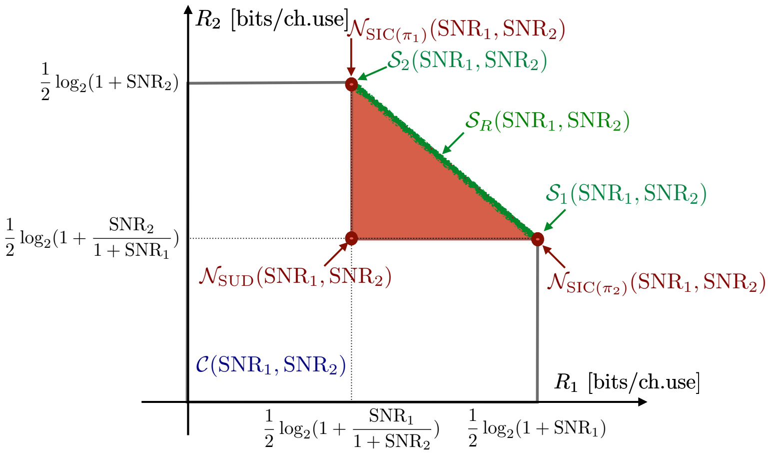

Figure 2: η-Nash and η-sequential equilibria regions, with $\eta \geqslant 0$ arbitrarily small, for the considered games in the two-user G-MAC with SNRs: SNR_1 and SNR_2. Here π_i refers to the decoding order in which transmitter i is decoded first, for all $i \in \{1, 2\}$.

The η-sequential equilibrium regions $\mathcal{S}_R(\mathrm{SNR}_1, \mathrm{SNR}_2)$, $\mathcal{S}_1(\mathrm{SNR}_1, \mathrm{SNR}_2)$, and $\mathcal{S}_2(\mathrm{SNR}_1, \mathrm{SNR}_2)$ in Theorems 4 and 5, respectively, are plotted in green.

Existence of η-NE and η-SE: For any nonnegative $\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K$, the existence of an η-NE and an η-SE, with η arbitrarily small, is always guaranteed. This statement follows immediately from the fact that the regions in Theorems 1-5 are nonempty. Note in particular that $\mathcal{N}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \neq \emptyset$ and $\mathcal{N}_{\mathrm{SIC}(\pi)}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \neq \emptyset$ for any $\pi \in \mathcal{P}_K$. Thus, $\mathcal{N}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \neq \emptyset$, which ensures the existence of at least one action profile $(\tilde{s}_0, s_1^*, \ldots, s_K^*)$ that is an η-NE, under any fixed receive strategy $\tilde{s}_0$.

Cardinality of η-NE and η-SE:: In both games $\mathcal{G}_1$ and $\mathcal{G}_2$ described in Secs. 2 and 3, the unicity of a given η-NE or η-SE of the considered game is not ensured even in the case in which the cardinality of the equilibrium region is one. This is mainly due to the fact that a given rate tuple can be achieved by various transmit and receive configurations. When the set of actions is more restricted, i.e., power control, then the unicity is ensured [9].

Optimality: In the first game in which only the transmitters are players, depending on the choice of the receiver, the η-NE rate tuples are not necessarily Pareto-optimal. On the other hand, in both sequential games, the η-SE rate tuples are Pareto-optimal. This suggests that, under the assumption that the players are able to properly choose the operating equilibrium action profiles for instance via learning algorithms, there is no loss of performance in the decentralized G-MAC case with respect to the fully centralized case. Furthermore, in both sequential games, the utility of the leader is always maximized, and thus it is always better to move first. Note that the definition of the sequential games in this report allows for a non-unitary set of leaders. Even though the analysis here is restricted only to the game with unitary sets of leaders, the above statement continues to hold for non-unitary sets of leaders.

Potential Games: The definition of the utilities of the transmitters and the receiver in (8) and (9), respectively, does not impose any restriction on the action sets, which can be complex objects. From this perspective, it is hard to cast the games presented here as potential games. If the actions of the players are restricted for instance to power allocation policies, the results on power allocation games in [9, 10, 11, 12, 13] can be seen as special cases of the results presented in this report.

A Proof of Theorem 1

Consider the set of all rate tuples that can be achieved under the assumption that the receiver performs SUD to recover the messages $M_1, M_2, \ldots, M_K$ and let $\tilde{s}_0$ denote the corresponding receive configuration that is fixed and known to everyone. This set is denoted by $\mathcal{C}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ and is given by:

$$\mathcal{C}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) = \left\{(R_1, R_2, \ldots, R_K) \in \mathcal{C}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) : R_i \leqslant \frac{1}{2}\log_2\left(1 + \frac{\mathrm{SNR}_i}{1 + \sum_{j=1; j\neq i}^{K} \mathrm{SNR}_j}\right), \forall i \in \mathcal{K}\right\}. \tag{27}$$

Let the subset $\mathcal{V}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ contain all nonnegative rate tuples $(R_1, R_2, \ldots, R_K) \in \mathcal{C}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ satisfying

$$R_i = \frac{1}{2}\log_2\left(1 + \frac{\mathrm{SNR}_i}{1 + \sum_{j=1; j\neq i}^{K} \mathrm{SNR}_j}\right), \quad \forall i \in \mathcal{K}. \tag{28}$$

Let also the subset $\bar{\mathcal{V}}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ be defined by $\bar{\mathcal{V}}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2 \ldots, \mathrm{SNR}_K) \triangleq \mathcal{C}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \backslash \mathcal{V}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$. Note that the sets $\mathcal{V}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ and $\bar{\mathcal{V}}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ form a partition of $\mathcal{C}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$.

Following this notation, the proof of Theorem 1 is established by Propositions 1 and 2.

Proposition 1. *Any rate tuple* $(R_1, R_2, \ldots, R_K) \in \bar{\mathcal{V}}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ *is not an η-NE with η arbitrarily small. That is,*

$$\mathcal{N}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \subseteq \mathcal{V}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K). \tag{29}$$

Proof: The proof of Proposition 1 is provided in Section A.1. ∎

Proposition 2. *Any rate tuple* $(R_1, R_2, \ldots, R_K) \in \mathcal{V}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ *is achievable at an η-NE with an arbitrarily small $\eta \geqslant 0$. That is,*

$$\mathcal{V}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \subseteq \mathcal{N}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K). \tag{30}$$

Proof: The proof of Proposition 2 is provided in Section A.2. ∎

A.1 Proof of Proposition 1

Any rate tuple $(R_1, R_2, \ldots, R_K) \in \bar{\mathcal{V}}_{\mathrm{SUD}}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ satisfies at least for one $i \in \{1, 2, \ldots, K\}$ the following condition:

$$R_i < \frac{1}{2}\log_2\left(1 + \frac{\mathrm{SNR}_i}{1 + \sum_{j=1; j\neq i}^{K} \mathrm{SNR}_j}\right). \tag{31}$$

Let $(R_1^*, R_2^*, \ldots, R_K^*)$ be an η-NE for any $\eta \geqslant 0$ arbitrarily small, achievable by an action profile $(\tilde{s}_0, s_1^*, \ldots, s_K^*) \in \mathcal{A}_0 \times \mathcal{A}_1 \times \cdots \times \mathcal{A}_K$. Denote by $X_{i,1}^*, X_{i,2}^*, \ldots, X_{i,n}^*$ the channel inputs generated by transmitter i, for $i \in \{1, 2, \ldots, K\}$, corresponding to the equilibrium action s_i^* and denote by $P_i^* \triangleq \frac{1}{n}\sum_{t=1}^{n} \mathrm{E}\left[(X_{i,t}^*)^2\right]$ their average power.

From the assumption that $(R_1^*, R_2^*, \dots, R_K^*)$ is achievable, $P_{\text{error}}^{(n)}(R_1^*, R_2^*, \dots, R_K^*)$ can be made arbitrarily small. Thus, from (8) it follows that

$$u_i(\tilde{s}_0, s_1^*, \dots, s_K^*) = R_i^*, \quad \forall i \in \{1, 2, \dots, K\}. \tag{32}$$

Using this notation, a necessary condition for η-NE action profiles is provided by Lemma 1.

Lemma 1 (IID Gaussian Inputs With Maximum Power)**.** *A necessary condition for the action profile $(\tilde{s}_0, s_1^*, \dots, s_K^*)$ to be an η-NE action is that the input symbols $X_{i,t}^*$, with $i \in \{1, 2, \dots, K\}$, must be generated i.i.d. following a zero-mean Gaussian distribution with variance $P_i^* = P_{i,\max}$.*

Proof: Without loss of generality, consider transmitter 1 and let $\tilde{R}_1$ denote the information rate that can be achieved by transmitter 1 when the input symbols are generated i.i.d. following a Gaussian distribution with maximum power $P_{1,\max}$ and where the channel inputs of all transmitters are uncorrelated.

Assume that in the action s_1^*, the input symbols are not generated i.i.d. following a Gaussian distribution with variance P_1^*. As the Gaussian distribution maximizes the entropy and as the information rates are increasing in the input power, using non-Gaussian inputs or using less power results in a loss in the achievable rate. Thus, in the action s_1^* the utility of transmitter 1 is $u_1(\tilde{s}_0, s_1^*, \dots, s_K^*) = R_1^* = \tilde{R}_1 - \zeta$, where $\zeta > 0$ quantifies the loss in transmitter 1's rate. From the assumption that the receiver implements SUD, independently of the actions $s_2^*, \dots, s_K^*$ of transmitters $2, 3, \dots, K$, there always exists an alternative action s_1 in which transmitter 1 uses i.i.d. Gaussian codebooks with variance $P_1^* = P_{1,\max}$, which achieves an information rate (and thus a utility) $u_1(\tilde{s}_0, s_1, s_2^*, \dots, s_K^*) = \tilde{R}_1$. Hence, it follows that

$$u_1(\tilde{s}_0, s_1, s_2^*, \dots, s_K^*) - u_1(\tilde{s}_0, s_1^*, s_2^*, \dots, s_K^*) = \zeta > 0. \tag{33}$$

The utility improvement is bounded away from zero, thus the action profile $(\tilde{s}_0, s_1^*, s_2^*, \dots, s_K^*)$ cannot be an η-NE (Def. 1), with an arbitrarily small $\eta \geqslant 0$. ■

Without loss of generality, consider transmitter 1 and assume that in the action profile $(\tilde{s}_0, s_1^*, \dots, s_K^*)$, the rate R_1^* satisfies (31) with $i = 1$. From Lemma 1, a necessary condition for the action s_1^* to be an η-NE action is to have i.i.d. Gaussian channel inputs with maximum power $P_{1,\max}$. This condition implies that any rate R_1 satisfying $0 \leqslant R_1 \leqslant \frac{1}{2}\log_2\left(1 + \frac{\text{SNR}_1}{1+\sum_{j=2}^{K}\text{SNR}_j}\right)$, can be achieved with arbitrarily small probability of error. Assume that in the action s_1^* transmitter 1's utility satisfies

$$u_1(\tilde{s}_0, s_1^*, \dots, s_K^*) = R_1^* = \frac{1}{2}\log_2\left(1 + \frac{\text{SNR}_1}{1 + \sum_{j=2}^{K}\text{SNR}_j}\right) - \xi, \tag{34}$$

with $\xi > 0$. Regardless of the action of the other transmitters, transmitter 1 can always choose an alternative action s_1' in which it has a utility

$$u_1(\tilde{s}_0, s_1', s_2^*, \dots, s_K^*) = \tilde{R}_1 = \frac{1}{2}\log_2\left(1 + \frac{\text{SNR}_1}{1 + \sum_{j=2}^{K}\text{SNR}_j}\right). \tag{35}$$

From (34) and (35), it holds that

$$u_1(\tilde{s}_0, s_1', s_2^*, \dots, s_K^*) - u_1(\tilde{s}_0, s_1^*, s_2^*, \dots, s_K^*) = \xi > 0. \tag{36}$$

The utility improvement is bounded away from zero, thus the action profile $(\tilde{s}_0, s_1^*, \dots, s_K^*)$ cannot be η-NE (Def. 1), with an arbitrarily small $\eta \geqslant 0$, which establishes the proof of Proposition 1.

A.2 Proof of Proposition 2

Let $\eta \geqslant 0$ be arbitrarily small and assume that the decoder performs SUD. To achieve the rate tuple $(R_1^*, R_2^*, \ldots, R_K^*)$ satisfying (28), transmitters $1, 2, \ldots, K$ can use the action profile $(\tilde{s}_0, s_1^*, \ldots, s_K^*)$ in which independent Gaussian codebooks are used with powers $P_{1,\max}, P_{2,\max}, \ldots, P_{K,\max}$, respectively, as in [1] or [2]. The messages $M_1, M_2, \ldots, M_K$ are encoded at the information rates $R_1^*, R_2^*, \ldots, R_K^*$, respectively. From the assumption that the receiver performs SUD, the probability of error $P_{\text{error}}^{(n)}(R_1^*, R_2^*, \ldots, R_K^*)$ can be made arbitrarily small as the block-length tends to infinity. Hence, the resulting utilities are given by $u_i(\tilde{s}_0, s_1^*, \ldots, s_K^*) = R_i^*, \quad i \in \{1, 2, \ldots, K\}$. Assume that the action profile $(\tilde{s}_0, s_1^*, \ldots, s_K^*)$ is not an η-NE. Then, from Def. 1, there exist at least one player $i \in \mathcal{K}$ and at least one strategy $s_i \in \mathcal{A}_i$ such that the utility u_i is improved by at least η bits per channel use when player i deviates from s_i^* to s_i. Without loss of generality, let transmitter 1 be the deviating player and denote by $\tilde{R}_1$ its new information rate. Hence,

$$u_1(\tilde{s}_0, s_1, \ldots, s_K^*) = \tilde{R}_1 > u_1(\tilde{s}_0, s_1^*, \ldots, s_K^*) + \eta, \tag{37}$$

and thus, it holds that $\tilde{R}_1 > \frac{1}{2}\log_2\left(1 + \frac{\text{SNR}_1}{1+\sum_{j=2}^{K}\text{SNR}_j}\right) + \eta$. As the rate tuple $(R_1^*, R_2^*, \ldots, R_K^*)$ already saturates the decoding capability of the receiver, the new rate tuple $(\tilde{R}_1, R_2^*, \ldots, R_K^*)$ cannot be achieved and will result in a probability of error bounded away from zero and consequently the corresponding utility will be $u_1(\tilde{s}_0, s_1, s_2^*, \ldots, s_K^*) = 0$, which contradicts the initial assumption (37) and establishes that the action profile $(\tilde{s}_0, s_1^*, \ldots, s_K^*)$ is an η-NE.

B Proof of Theorem 2

The proof of Theorem 2 follows along the same lines as the proof of Theorem 1 when considering the set $\mathcal{C}_{\text{SIC}(\pi)}(\text{SNR}_1, \text{SNR}_2, \ldots, \text{SNR}_K)$. This set contains all rate tuples $(R_1, R_2, \ldots, R_K)$ which can be achieved if the receiver performs SIC(π). It is given by

$$\mathcal{C}_{\text{SIC}(\pi)}(\text{SNR}_1, \text{SNR}_2, \ldots, \text{SNR}_K) = \left\{(R_1, R_2, \ldots, R_K) \in \mathcal{C}(\text{SNR}_1, \text{SNR}_2, \ldots, \text{SNR}_K) : R_{\pi(i)} \leqslant \frac{1}{2}\log_2\left(1 + \frac{\text{SNR}_{\pi(i)}}{1 + \sum_{j=i+1}^{K}\text{SNR}_{\pi(j)}}\right), \forall i \in \mathcal{K}\right\}. \tag{38}$$

C Proof of Theorem 4

Let the set $\mathcal{W}_R(\text{SNR}_1, \text{SNR}_2, \ldots, \text{SNR}_K)$ be defined as follows:

$$\mathcal{W}_R(\text{SNR}_1, \text{SNR}_2, \ldots, \text{SNR}_K) \triangleq \left\{(R_1, R_2, \ldots, R_K) \in \mathcal{C}(\text{SNR}_1, \text{SNR}_2, \ldots, \text{SNR}_K) \text{ s.t. } \sum_{i=1}^{K} R_i = \frac{1}{2}\log_2\left(1 + \sum_{i=1}^{K}\text{SNR}_i\right)\right\}. \tag{39}$$

Let also $\bar{\mathcal{W}}_R(\text{SNR}_1, \text{SNR}_2, \ldots, \text{SNR}_K)$ be defined as

$$\bar{\mathcal{W}}_R(\text{SNR}_1, \text{SNR}_2, \ldots, \text{SNR}_K) \triangleq \mathcal{C}(\text{SNR}_1, \text{SNR}_2, \ldots, \text{SNR}_K) \backslash \mathcal{W}_R(\text{SNR}_1, \text{SNR}_2, \ldots, \text{SNR}_K). \tag{40}$$

That is, $\mathcal{W}_R(\text{SNR}_1, \text{SNR}_2, \ldots, \text{SNR}_K)$ contains all rate tuples $(R_1, R_2, \ldots, R_K)$ which saturate the sum-capacity of the channel.

The proof of Theorem 4 is established by Propositions 3 and 4.

Proposition 3 (Non-Equilibrium Points)**.** *Any rate tuple* $(R_1, R_2, \ldots, R_K) \in \bar{\mathcal{W}}_R(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ *is not an* η*-SE with* η *arbitrary small. That is,*

$$\mathcal{S}_R(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \subseteq \mathcal{W}_R(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K). \quad (41)$$

Proposition 4 (Achievability)**.** *Any rate tuple* $(R_1, R_2, \ldots, R_K) \in \mathcal{W}_R(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ *is achievable at an* η*-SE with* η *arbitrary small. That is,*

$$\mathcal{W}_R(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \subseteq \mathcal{S}_R(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K). \quad (42)$$

C.1 Proof of Proposition 3

Let $(R_1^\dagger, R_2^\dagger, \ldots, R_K^\dagger)$ be an η-SE for any $\eta \geqslant 0$ arbitrarily small, achievable by an action profile $(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger) \in \mathcal{A}_0 \times \mathcal{A}_1 \times \cdots \times \mathcal{A}_K$.

Denote by $X_{i,1}^\dagger, X_{i,2}^\dagger, \ldots, X_{i,n}^\dagger$ the channel inputs generated by transmitter i, for $i \in \{1, 2, \ldots, K\}$, corresponding to the equilibrium action $s_i^\dagger$ and denote by $P_i^\dagger \triangleq \frac{1}{n}\sum_{t=1}^{n} \mathrm{E}\left[(X_{i,t}^\dagger)^2\right]$ their average power.

From the assumption that $(R_1^\dagger, R_2^\dagger, \ldots, R_K^\dagger)$ is achievable, $P_{\mathrm{error}}^{(n)}(R_1^\dagger, R_2^\dagger, \ldots, R_K^\dagger)$ can be made arbitrarily small. Thus, from (8) and (9) it follows that

$$u_i(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger) = R_i^\dagger, \quad \forall i \in \{1, 2, \ldots, K\}, \quad (43)$$

$$u_0(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger) = \sum_{j=1}^{K} R_j^\dagger. \quad (44)$$

Any action profile $(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger)$ in which the transmitters use information rates $(R_1^\dagger, R_2^\dagger, \ldots, R_K^\dagger)$ which do not saturate $R_{\mathrm{sum}}(s_0^\dagger)$, i.e. $\sum_{i=1}^{K} R_i^\dagger < R_{\mathrm{sum}}(s_0^\dagger)$, cannot correspond to an η-SE. This is mainly due to the fact that at least one of the transmitters can always increase its rate by a positive δ and ensure a utility improvement that is bounded away from zero, which contradicts the assumption of an η-NE among the followers knowing the leader's strategy, and thus the action profile $(s_0^\dagger, s_1^\dagger, s_2^\dagger, \ldots, s_K^\dagger)$ cannot be an η-SE. Hence, the utility the receiver's utility at the η-SE action profile $(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger)$, is given by

$$u_0(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger) = R_{\mathrm{sum}}(s_0^\dagger). \quad (45)$$

Assume now that in the action $s_0^\dagger$, the receiver uses a decoding strategy under which the maximum sum-rate that can be achieved, $R_{\mathrm{sum}}(s_0^\dagger)$, satisfies

$$R_{\mathrm{sum}}(s_0^\dagger) = \frac{1}{2}\log_2\left(1 + \sum_{i=1}^{K} \mathrm{SNR}_i\right) - \xi, \quad (46)$$

with $\xi > 0$. There always exists an alternative action of the receiver $\tilde{s}_0$ for which

$$R_{\mathrm{sum}}(\tilde{s}_0) = \frac{1}{2}\log_2\left(1 + \sum_{i=1}^{K} \mathrm{SNR}_i\right). \quad (47)$$

The transmitters adapt to the leader's action by transmitting at rates which saturate the new sum-rate $R_{\mathrm{sum}}(s_0^\dagger)$. Hence, it follows that

$$u_0(\tilde{s}_0, s_1^\dagger, \ldots, s_K^\dagger) - u_1(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger) = \xi > 0. \quad (48)$$

The utility improvement is bounded away from zero, thus the action profile $(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger)$ cannot be η-SE (Def. 4), with an arbitrarily small $\eta \geqslant 0$.

C.2 Proof of Proposition 4

Assume a given ordering of the permutations over $\{1, 2, \ldots, K\}$ such that the set $\mathcal{P}_K$ can be written as $\mathcal{P}_K \triangleq \{\pi_j\}_{j=1}^{K!}$.

For all $j \in \{1, 2, \ldots, K!\}$, let A_j denote the rate point with coordinates

$$R_{\pi_j(i)} = \frac{1}{2}\log_2\left(1 + \frac{\mathrm{SNR}_{\pi_j(i)}}{1 + \sum_{\ell=i+1}^{K}\mathrm{SNR}_{\pi_j(\ell)}}\right), \forall i \in \{1, 2, \ldots, K\}. \tag{49}$$

Any rate tuple $(R_1, R_2, \ldots, R_K) \in \mathcal{W}_R(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ can be written as

$$R_i = \sum_{j=1}^{K!} \lambda_j R_{i,A_j}, \quad i \in \{1, 2, \ldots, K\}, \tag{50}$$

for some real nonnegative parameters $(\lambda_1, \lambda_2, \ldots, \lambda_{K!})$ satisfying $\sum_{j=1}^{K!} \lambda_j = 1$ and where R_{i,A_j} denotes the rate of transmitter i in the coordinates of A_j.

Fix a tuple $(\lambda_{1,Q}, \lambda_{2,Q}, \ldots, \lambda_{K!,Q})$ satisfying $\sum_{j=1}^{K!} \lambda_{j,Q} = 1$ and consider the rate point Q with coordinates

$$R_{i,Q} = \sum_{j=1}^{K!} \lambda_{j,Q} R_{i,A_j}, \quad i \in \{1, 2, \ldots, K\}. \tag{51}$$

The point Q is achievable if the transmitters and the receiver use the action profile $(s_{0,Q}, s_{1,Q}, \ldots, s_{K,Q})$ described as follows. The receiver who is the leader chooses the action $s_{0,Q}$ in which it performs a time-sharing between the successive interference cancellation decoding orders $\mathrm{SIC}(\pi_1), \mathrm{SIC}(\pi_2), \ldots, \mathrm{SIC}(\pi_{K!})$ with time-sharing parameters $\lambda_{1,Q}, \lambda_{2,Q}, \ldots, \lambda_{K!,Q}$. That is, during the first $n\lambda_{1,Q}$ channel uses, it performs $\mathrm{SIC}(\pi_1)$, during the following $n\lambda_{2,Q}$ channel uses it performs $\mathrm{SIC}(\pi_2)$, etc. The transmitters follow the receiver and use i.i.d. Gaussian codebooks with the rates $(R_{1,A_1}, R_{2,A_1}, \ldots, R_{K,A_1})$, as in [1] or [2], during the first $n\lambda_{1,Q}$ channel uses, and with rates $(R_{1,A_2}, R_{2,A_2}, \ldots, R_{K,A_2})$ during the following $n\lambda_{2,Q}$ channel uses, etc. Since the overall probability of error $P_{\mathrm{error}}^{(n)}(R_{1,Q}, R_{2,Q}, \ldots, R_{K,Q})$ is less than the sum of the probabilities of error of each of the $K!$ segments, $P_{\mathrm{error}}^{(n)}(R_{1,Q}, R_{2,Q}, \ldots, R_{K,Q})$ can be made arbitrarily small as the blocklength goes to infinity and the resulting utilities are given by

$$u_i(s_{1,Q}, s_{2,Q}, \ldots, s_{K,Q}) = R_{i,Q}, \quad i \in \{1, 2, \ldots, K\}, \tag{52}$$

$$u_0(s_{1,Q}, s_{2,Q}, \ldots, s_{K,Q}) = \sum_{i=1}^{K} R_{i,Q}, \tag{53}$$

Assume that the action profile $(s_{0,Q}, s_{1,Q} \ldots, s_{K,Q})$ is not an η-SE. Then, from Def. 4, there exist at least one player $i \in \{0, 1, \ldots, K\}$ and at least one strategy $s_i \in \mathcal{A}_i$ such that the utility u_i is improved by at least η bits per channel use when player i deviates from $s_{i,Q}$ to s_i.

Two cases are to be considered. Either the deviating player is one of the transmitters or the deviating player is the receiver.

Consider first the case in which the deviating player is one of the transmitters. Assume without loss of generality that the deviating player is transmitter 1 and denote by $\tilde{R}_1$ its new information rate in the new action s_1. Hence,

$$u_0(s_{0,Q}, s_1, \ldots, s_{K,Q}) = R_1 > u_0(s_{0,Q}, s_{1,Q}, \ldots, s_{2,Q}) + \eta. \tag{54}$$

Since the rate tuple $(R_{1,Q}, R_{2,Q}, \ldots, R_{K,Q})$ already saturates the decoding capability of the receiver, the new rate tuple $(R_1, R_{2,Q}, \ldots, R_{K,Q})$ cannot be achieved and will result in a probability of error bounded away from zero. Consequently, the corresponding utility will be $u_1(s_{0,Q}, s_1, \ldots, s_{K,Q}) = 0$, which contradicts the initial assumption (54).

Now consider the case in which the deviating player is the receiver. Let s_0 denote its new action and assume that its corresponding utility is

$$u_0(s_0, s_{1,Q}, \ldots, s_{K,Q}) = R_{\mathrm{sum}}(s_0) > u_0(s_{0,Q}, s_{1,Q}, \ldots, s_{K,Q}) + \eta. \tag{55}$$

As the action profile $(s_{0,Q}, s_{1,Q} \ldots, s_{K,Q})$ already saturates the sum-capacity, for all possible $s_0 \in \mathcal{A}_0$, the utility of the receiver satisfies

$$u_0(s_0, s_{1,Q}, \ldots, s_{K,Q}) \leqslant u_0(s_{0,Q}, s_{1,Q}, \ldots, s_{K,Q}) + \eta. \tag{56}$$

which contradicts the assumption (55) and establishes the proof.

D Proof of Theorem 5

Let the set $\mathcal{W}_i(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ be defined as follows:

$$\begin{aligned}&\mathcal{W}_i(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)\\ &\triangleq \Bigg\{(R_1, R_2, \ldots, R_K) \in \mathcal{C}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \text{ s.t. } R_i = \frac{1}{2}\log_2(1+\mathrm{SNR}_i) \text{ and}\\ &\qquad \sum_{j=1; j\neq i}^{K} R_j = \frac{1}{2}\log_2\left(1+\sum_{j=1}^{K}\mathrm{SNR}_j\right) - \frac{1}{2}\log_2(1+\mathrm{SNR}_i)\Bigg\}.\end{aligned} \tag{57}$$

Let also $\bar{\mathcal{W}}_i(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ be defined as

$$\bar{\mathcal{W}}_i(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \triangleq \mathcal{C}(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \backslash \mathcal{W}_i(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K). \tag{58}$$

The proof of Theorem 5 is established by Propositions 5 and 6.

Proposition 5 (Non-Equilibrium Points)**.** *Any rate tuple* $(R_1, R_2, \ldots, R_K) \in \bar{\mathcal{W}}_i(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ *is not an* η*-SE with* η *arbitrary small. That is,*

$$\mathcal{S}_i(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \subseteq \mathcal{W}_i(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K). \tag{59}$$

Proposition 6 (Achievability)**.** *Any rate tuple* $(R_1, R_2, \ldots, R_K) \in \mathcal{W}_i(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ *is achievable at an* η*-SE with* η *arbitrary small. That is,*

$$\mathcal{W}_i(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K) \subseteq \mathcal{S}_i(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K). \tag{60}$$

D.1 Proof of Proposition 5

Let $(R_1^\dagger, R_2^\dagger, \ldots, R_K^\dagger)$ be an η-SE for any $\eta \geqslant 0$ arbitrarily small, achievable by an action profile $(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger) \in \mathcal{A}_0 \times \mathcal{A}_1 \times \cdots \times \mathcal{A}_K$.

Denote by $X_{i,1}^\dagger, X_{i,2}^\dagger, \ldots, X_{i,n}^\dagger$ the channel inputs generated by transmitter i, for $i \in \{1, 2, \ldots, K\}$, corresponding to the equilibrium action $s_i^\dagger$ and denote by $P_i^\dagger \triangleq \frac{1}{n}\sum_{t=1}^{n} \mathrm{E}\left[(X_{i,t}^\dagger)^2\right]$ their average power.

From the assumption that $(R_1^\dagger, R_2^\dagger, \ldots, R_K^\dagger)$ is achievable, the probability of error $P_{\mathrm{error}}^{(n)}(R_1^\dagger, R_2^\dagger, \ldots, R_K^\dagger)$ can be made arbitrarily small, and thus from (8) and (9) it follows that

$$u_i(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger) = R_i^\dagger, \quad \forall i \in \{1, 2, \ldots, K\}, \tag{61}$$

$$u_0(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger) = \sum_{j=1}^{K} R_j^\dagger. \tag{62}$$

Consider transmitter i and assume that in the action $s_i^\dagger$, the rate $R_i^\dagger$ satisfies

$$R_i^\dagger \leqslant \frac{1}{2}\log_2(1 + \mathrm{SNR}_i) - \delta, \tag{63}$$

with $\delta > 0$.

There always exist an alternative action $\tilde{s}_i$ of transmitter i which achieves a rate

$$R_i = \frac{1}{2}\log_2(1 + \mathrm{SNR}_i). \tag{64}$$

Hence, for sufficiently large n, the leader's utility improvement is bounded away from zero and the action profile $(s_0^\dagger, s_1^\dagger, \ldots, s_K^\dagger)$ cannot be η-SE (Def. 4), with an arbitrarily small $\eta \geqslant 0$.

Following a similar reasoning as in the proof of Theorem 4, a necessary condition for an action profile to be an η-SE, with $\eta \geqslant 0$ arbitrarily small, is to have a decoding strategy at the receiver which allows the achievability of the sum-capacity. That is,

$$R_{\mathrm{sum}}(s_0^\dagger) = \sum_{j=1}^{K} R_j^\dagger = \frac{1}{2}\log_2\left(1 + \sum_{j=1}^{K}\mathrm{SNR}_j\right). \tag{65}$$

Now assume that in the action profile $(s_0^\dagger, \ldots, s_K^\dagger)$,

$$R_i^\dagger = \frac{1}{2}\log_2(1 + \mathrm{SNR}_i), \tag{66}$$

and

$$\sum_{j=1; j\neq i}^{K} R_j^\dagger - \frac{1}{2}\log_2\left(1 + \sum_{j=1}^{K}\mathrm{SNR}_j\right) - \frac{1}{2}\log_2(1 + \mathrm{SNR}_i) - \xi. \tag{67}$$

In this case, at least one transmitter j, with $j \neq i$, can always increase its information rate, and thus its utility by a $\mu > 0$ while guaranteeing the achievability of the rate-tuple $(R_j^\dagger + \mu, \mathbf{R}_{-j}^\dagger)$ as long as

$$\mu \leqslant \min\left\{\xi, \frac{1}{2}\log_2(1 + \mathrm{SNR}_j) - R_j^\dagger\right\}. \tag{68}$$

The utility improvement for this player is bounded away from zero; and thus the action profile $(s_0^\dagger, \ldots, s_K^\dagger)$ cannot be η-SE (Def. 4), with an arbitrarily small $\eta \geqslant 0$.

D.2 Proof of Proposition 6

Let $\mathcal{P}_{i,K}$ be a set of containing all permutations $\kappa \in \mathcal{P}_K$ such that $\kappa(K) = i$. The cardinality of this set is $(K-1)!$. Assume a given ordering of these permutations such that the set $\mathcal{P}_{i,K}$ can be written as $\mathcal{P}_{i,K} \triangleq \{\kappa_j\}_{j=1}^{(K-1)!}$.

For all $j \in \{1, 2, \ldots, (K-1)!\}$, let B_j denote the rate point with coordinates

$$R_i = \frac{1}{2}\log_2\left(1 + \mathrm{SNR}_i\right) \tag{69}$$

$$R_{\kappa_j(k)} = \frac{1}{2}\log_2\left(1 + \frac{\mathrm{SNR}_{\kappa_j(k)}}{1 + \sum_{\ell=k+1}^{K} \mathrm{SNR}_{\kappa_j(\ell)}}\right), \forall k \in \{1, 2, \ldots, K\} \setminus \{i\}. \tag{70}$$

Any rate tuple $(R_1, R_2, \ldots, R_K) \in \mathcal{W}_i(\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_K)$ can be written as

$$R_i = \frac{1}{2}\log_2\left(1 + \mathrm{SNR}_i\right) \tag{71}$$

$$R_k = \sum_{j=1}^{(K-1)!} \beta_j R_{k,B_j}, \quad k \in \{1, 2, \ldots, K\} \setminus \{i\}, \tag{72}$$

for some real nonnegative parameters $(\beta_1, \beta_2, \ldots, \beta_{(K-1)!})$ satisfying $\sum_{j=1}^{(K-1)!} \beta_j = 1$ and where R_{k,B_j} denotes the rate of transmitter k in the coordinates of B_j.

Fix a tuple $(\beta_{1,Q'}, \beta_{2,Q'}, \ldots, \lambda_{(K-1)!,Q'})$ satisfying $\sum_{j=1}^{(K-1)!} \beta_{j,Q'} = 1$ and consider the rate point Q' with coordinates

$$R_{i,Q'} = \frac{1}{2}\log_2\left(1 + \mathrm{SNR}_i\right), \tag{73}$$

$$R_{k,Q'} = \sum_{j=1}^{(K-1)!} \beta_{j,Q'} R_{k,B_j}, \quad k \in \{1, 2, \ldots, K\} \setminus \{i\}. \tag{74}$$

The point Q' is achievable if the transmitters and the receiver use the action profile $(s_{0,Q'}, s_{1,Q'}, \ldots, s_{K,Q'})$ described as follows. Transmitter i who is the leader chooses the action $s_{i,Q'}$ in which it transmits information at its maximum rate $R_i = \frac{1}{2}\log_2\left(1 + \mathrm{SNR}_i\right)$ using an iid Gaussian codebook with power $P_{i,\max}$. In order to adapt to the leader's choice, the receiver chooses a time-sharing between $\mathrm{SIC}(\kappa_1), \mathrm{SIC}(\kappa_2), \ldots, \mathrm{SIC}(\kappa_{(K-1)!})$ with time-sharing parameters $\beta_{1,Q'}, \beta_{2,Q'}, \ldots, \beta_{(K-1)!,Q'}$. The remaining transmitters use i.i.d. Gaussian codebooks with their maximum powers and with the adequate time-sharing rates to achieve the point Q'. As the overall probability of error $P_{\mathrm{error}}^{(n)}(R_{1,Q'}, R_{2,Q'}, \ldots, R_{K,Q'})$ is less than the sum of the probabilities of error of each of the $(K-1)!$ segments, $P_{\mathrm{error}}^{(n)}(R_{1,Q'}, R_{2,Q'}, \ldots, R_{K,Q'})$ can be made arbitrarily small as the blocklength goes to infinity and the resulting utilities are given by

$$u_k(s_{1,Q'}, s_{2,Q'}, \ldots, s_{K,Q'}) = R_{k,Q'}, \quad k \in \{1, 2, \ldots, K\}, \tag{75}$$

$$u_0(s_{1,Q'}, s_{2,Q'}, \ldots, s_{K,Q'}) = \sum_{k=1}^{K} R_{k,Q'}, \tag{76}$$

Assume that the action profile $(s_{0,Q'}, s_{1,Q'} \ldots, s_{K,Q'})$ is not an η-SE. Then, from Def. 4, there exist at least one player $k \in \{0, 1, \ldots, K\}$ and at least one strategy $s_k \in \mathcal{A}_k$ such that the utility u_k is improved by at least η bits per channel use when player k deviates from $s_{k,Q'}$ to s_k. Two cases are to be considered. The first case is to have the leader, Transmitter i, deviating from $s_{i,Q'}$. Let $\tilde{s}_i$ denote its new strategy in which it achieves a rate, and thus utility

$$u_i(s_{0,Q'}, \ldots, s_{i-1,Q'}, \tilde{s}_i, s_{i+1,Q'}, \ldots, s_{K,Q'}) = \tilde{R}_i > u_i(s_{0,Q'}, \ldots, s_{i-1,Q'}, s_{i,Q'}, s_{i+1,Q'}, \ldots, s_{K,Q'}) + \eta. \tag{77}$$

In the action $s_{i,Q'}$, transmitter i saturates its maximum individual rate and any information rate that is strictly larger cannot be achieved and would result in an error probability bounded away from zero, and thus the corresponding utility is zero, which contradicts (77).

The second case to be considered here is to have one of the followers deviating. If the deviating player is the receiver, using a similar argument as in the proof of Proposition 4, any unilateral deviation of the receiver will result in a loss in the sum-rate and cannot be an η-NE among the followers, and thus it cannot be an η-SE. If the deviating follower is transmitter j, as the rate tuple already saturates the sum-capacity, any deviation will result in an error probability that is bounded away from zero and thus a zero utility. This contradicts the assumption of an η-SE and establishes the proof of Proposition 6.

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Publisher
Inria
Domaine de Voluceau - Rocquencourt
BP 105 - 78153 Le Chesnay Cedex
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ISSN 0249-6399